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HIGH-ALTITUDE REENTRY DRAG EFFECTS

JUNE 1966

S. Sharenson

Prepared for

DEPUTY FOR ENGINEERING & TECHNOLOGY
DIRECTORATE OF RADAR & OPTICS
ELECTRONIC SYSTEMS DIVISION
AIR FORCE SYSTEMS COMMAND
UNITED STATES AIR FORCE
L. G. Hanscom Field, Bedford, Massachusetts

Advanced Research Projects Agency
Project Defender
ARPA Order No. 596

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Project 8090

Prepared by

THE MITRE CORPORATION
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Contract AF19(628)-5165

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FOREWORD

This research was supported by the Advanced Research Projects Agency (Project Defender), and was monitored by ESD under Contract No. AF 19(628)-5165.

REVIEW AND APPROVAL

Publication of this technical report does not constitute Air Force approval of the report's findings or conclusions. It is published only for the exchange and stimulation of ideas.

R. J. McConnell
for H. M. BYRAM
Project Officer

ABSTRACT

The effect of atmospheric drag on the trajectory of a reentry body is considered. The analysis is valid in the high altitude regime. Results are presented as the differential displacement and velocity from the vacuum trajectory for various values of ballistic coefficient.

LIST OF ILLUSTRATIONS

TABLE OF SYMBOLS AND UNITS

h = altitude - feet

$\rho(h)$ = atmospheric density - pounds per cubic foot

S = distance along trajectory - feet

v = velocity along trajectory - feet per second

β = ballistic coefficient - pounds per square foot

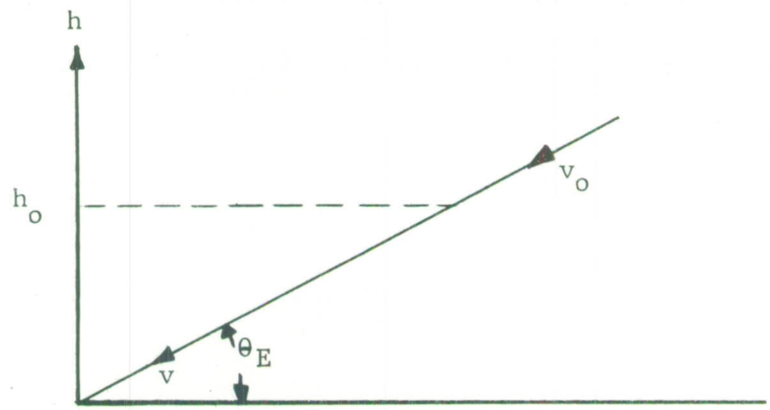
θ_E = reentry angle

t = time - seconds

The differential displacement of a re-entering body due to atmospheric drag is considered in this report. The situation considered is shown below. The body enters the atmosphere at an angle θ_E from the horizontal with an initial velocity v_o. The altitude, measured from the earth's surface, is h. The altitude h_o is chosen large enough such that:

$$\rho(h_o) \approx 0 \tag{1}$$

where $\rho(h)$ is the atmospheric density as a function of altitude.



Neglecting the earth's curvature, the effect of earth gravity and atmospheric drag, the trajectory of the re-entering body is a straight line and the displacement along the trajectory is given by:

$$S = \frac{h_o - h}{\sin \theta_E} \tag{2}$$

where

$$S = v_o t \tag{3}$$

Including, now, the effect of drag (but not lift), the classical equation for deceleration due to drag is:

$$-\frac{dv}{dt} = \frac{1}{2\beta} \rho(h) v^2 \tag{4}$$

Here v is the instantaneous velocity of the vehicle and β the ballistic coefficient.

Constraining the analysis to the high altitude re-entry regime where changes in v are small, the drag force can be calculated by assuming $v \approx v_o$ in this regime.

From equations (2) and (3):

$$dt = -\frac{dh}{v_o \sin \theta_E} \tag{5}$$

Substituting equation (4) for dt, and setting $v \approx v_o$:

$$\int_{v_o}^{v} dv = \int_{h_o}^{h} \frac{v_o}{2\beta \sin \theta_E} \rho(h)\, dh \tag{6}$$

$$v - v_o = \frac{v_o}{2\beta \sin \theta_E} \int_{h_o}^{h} \rho(h)\, dh \tag{7}$$

$$v = v_o + \frac{v_o}{2\beta \sin \theta_E} \int_{h_o}^{h} \rho(h)\, dh \tag{8}$$

But:

$$v = \frac{dS}{dt} \tag{9}$$

Therefore:

$$dS = v_o dt + \frac{v_o dt}{2\beta \sin \theta_E} \int_{h_o}^{h} \rho(h)\ dh \tag{10}$$

from equation (8). Substituting from equation (5):

$$dS = v_o dt - \frac{dh}{2\beta(\sin \theta_E)^2} \int_{h_o}^{h} \rho(h)\ dh \tag{11}$$

Integrating equation (11):

$$\int_{S_o}^{S} dS = v_o \int_{t_o}^{t} dt - \frac{1}{2\beta(\sin \theta_E)^2} \int_{h_o}^{h} dh' \int_{h_o}^{h'} \rho(h')\ dh' \tag{12}$$

But:

$$S_o = 0 \tag{13}$$

at $h = h_o$. Thus

$$S = v_o(t - t_o) - \frac{1}{2\beta(\sin \theta_E)^2} \int_{h_o}^{h} dh' \int_{h_o}^{h'} \rho(h')\ dh' \tag{14}$$

Now, in the absence of the atmosphere, equation (4) reduces to:

$$\frac{dv}{dt} = 0 \tag{15}$$

and the displacement is simply:

$$S' = v_o(t - t_o) \tag{16}$$

Subtracting (14) from (16) gives the differential displacement ΔS:

$$\Delta S = S' - S = \frac{1}{2\beta(\sin\theta_E)^2} \int_{h_o}^{h} dh' \int_{h_o}^{h'} \rho(h')\, dh' \tag{17}$$

For comparison the velocity, $\frac{d(\Delta S)}{dt}$, and the deceleration, $\frac{d^2(\Delta S)}{dt^2}$, are derived:

$$\frac{d(\Delta S)}{dt} = \frac{1}{2\beta(\sin\theta_E)^2} \frac{dh}{dt} \int_{h_o}^{h} \rho(h)\, dh \tag{18}$$

From equations (2) and (3):

$$\frac{dh}{dt} = - \sin\theta_E \frac{dS}{dt} \tag{19}$$

or

$$\frac{dh}{dt} = - \sin\theta_E\, v_o \tag{20}$$

Therefore:

$$\frac{d(\Delta S)}{dt} = - \frac{v_o}{2\beta\sin\theta_E} \int_{h_o}^{h} \rho(h)\, dh \tag{21}$$

But:

$$\int_{h_o}^{h} \rho(h)\, dh < 0 \tag{22}$$

Therefore:

$$\frac{d(\Delta S)}{dt} = \frac{v_o}{2\beta \sin \theta_E} \left| \int_{h_o}^{h} \rho(h)\ dh \right| \tag{23}$$

The deceleration is found from a second differentiation:

$$\frac{d^2(\Delta S)}{dt^2} = \frac{v_o}{2\beta \sin \theta_E} \frac{dh}{dt} \rho(h) \tag{24}$$

or:

$$\frac{d^2(\Delta S)}{dt^2} = - \frac{v_o^2}{2\beta} \rho(h) \tag{25}$$

The results of this analysis are presented in Figure 1, as ΔS vs. h for three different values of β. For comparison, $\frac{d(\Delta S)}{dt}$ and $\frac{d^2(\Delta S)}{dt^2}$ are presented in Figures 2 through 3. Since this analysis applies only in the region where the velocity change is small, the curves are terminated at the appropriate lowest altitude for the particular value of β. The atmospheric density profile used is known in Figure 4*. The first and second integrals of the density profile are given in Figures 5 and 6.

S. Sharenson

S. Sharenson

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*P. J. Nawrocki and R. Papa, Atmospheric Processes, Prentice-Hall, Inc., New Jersey, 1963.

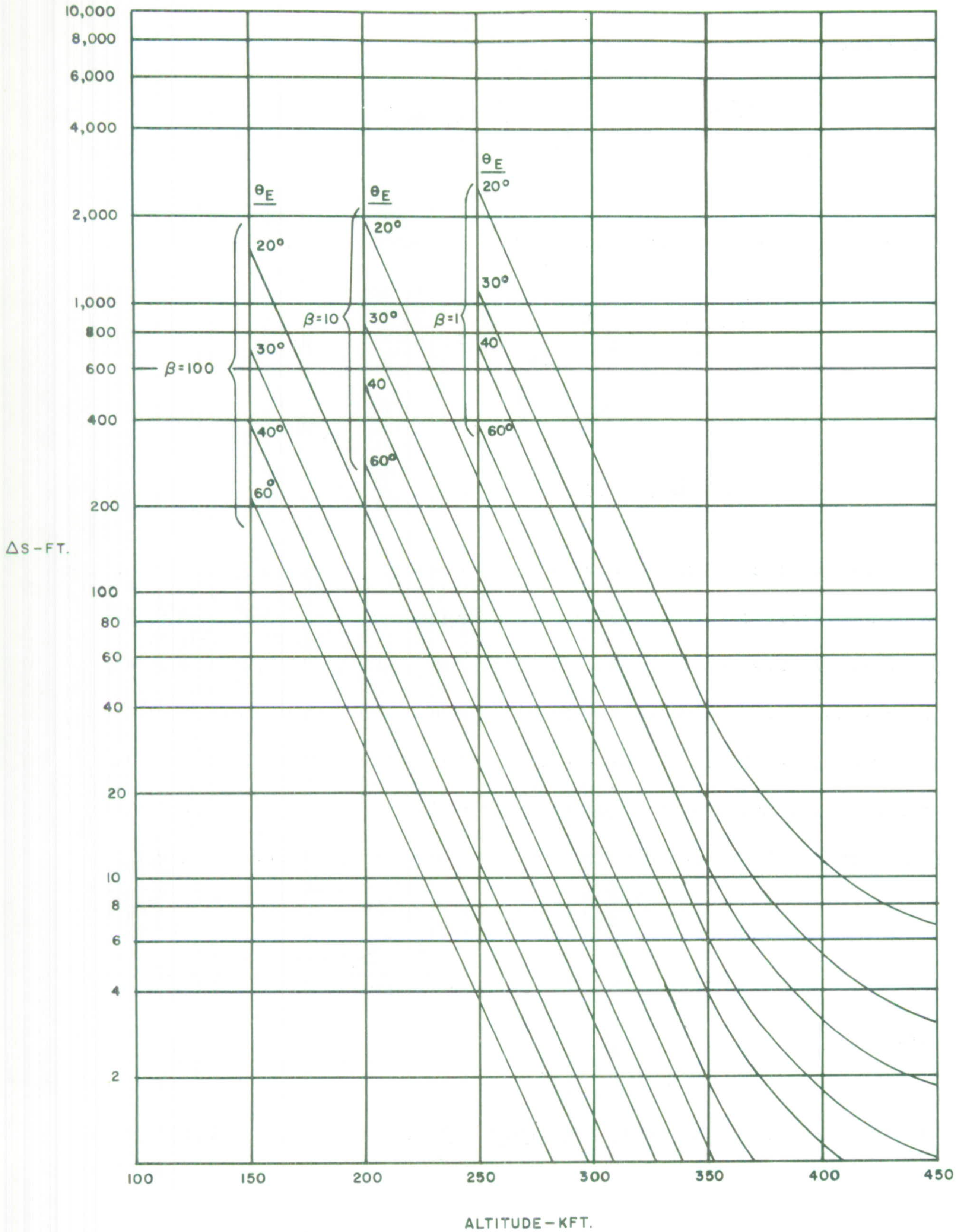

Figure 1

ΔS VS ALTITUDE θ = REENTRY ANGLE β = BALLISTIC COEFF. -PSF

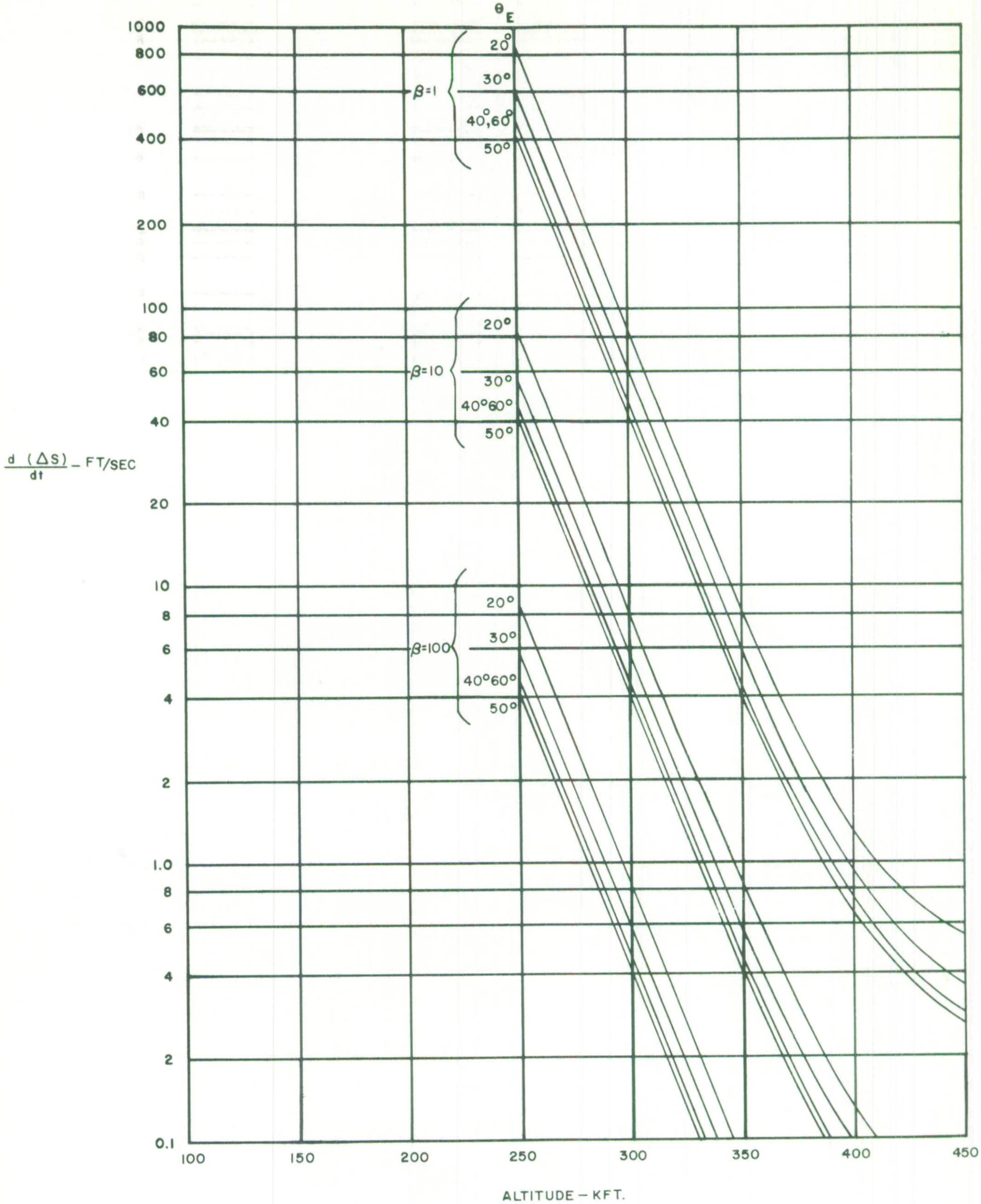

Figure 2

$\frac{d(\Delta S)}{dt}$ VS. ALTITUDE θ_E = REENTRY ANGLE

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5500 NM TRAJECTORY β = BALLISTIC COEFF. - PSF

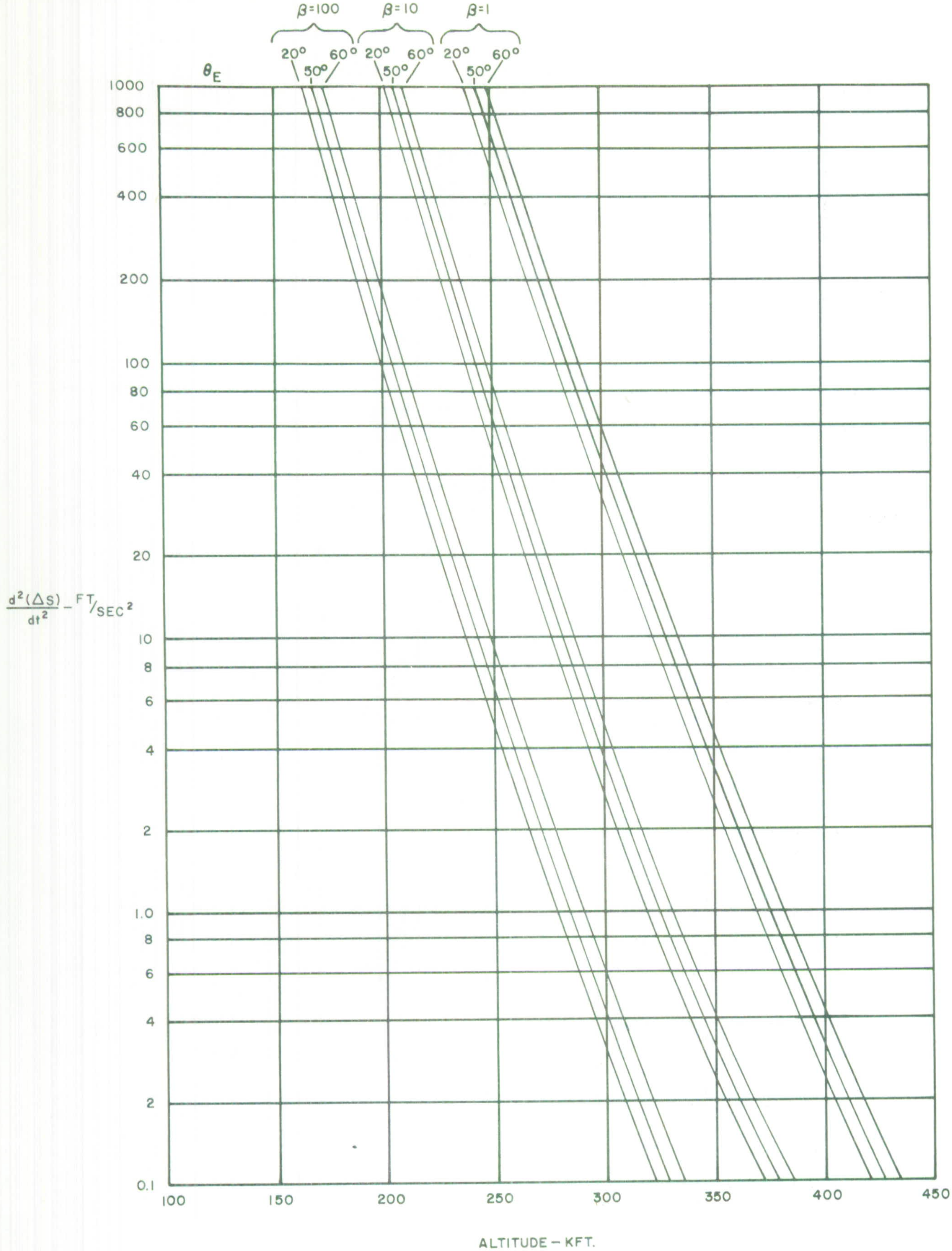

Figure 3

$\frac{d^2(\Delta S)}{dt^2}$ VS ALTITUDE θ_E = REENTRY ANGLE

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5500 NM TRAJECTORY β = BALLISTIC COEFF. – PSF

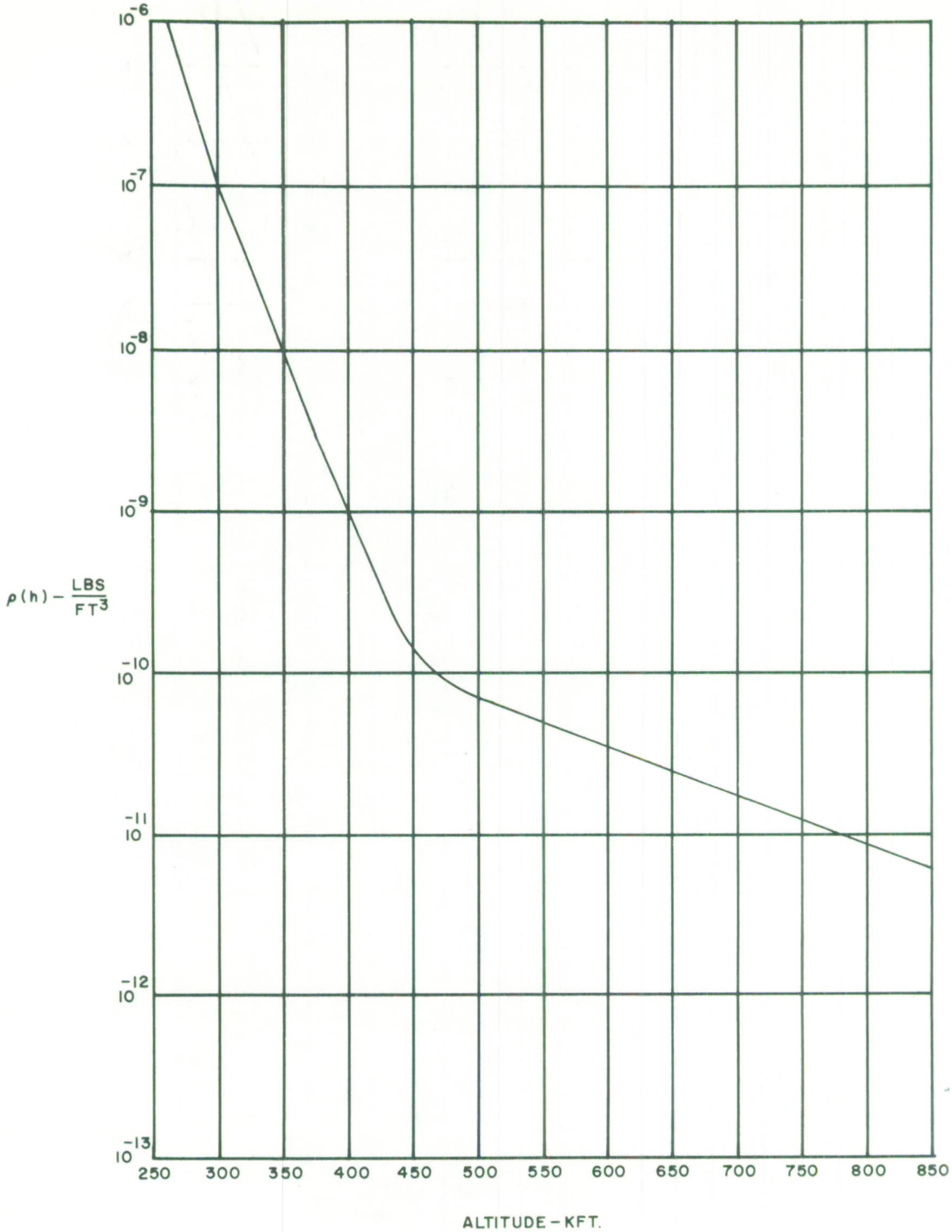

Figure 4

ATMOSPHERIC DENSITY - $\rho(h)$

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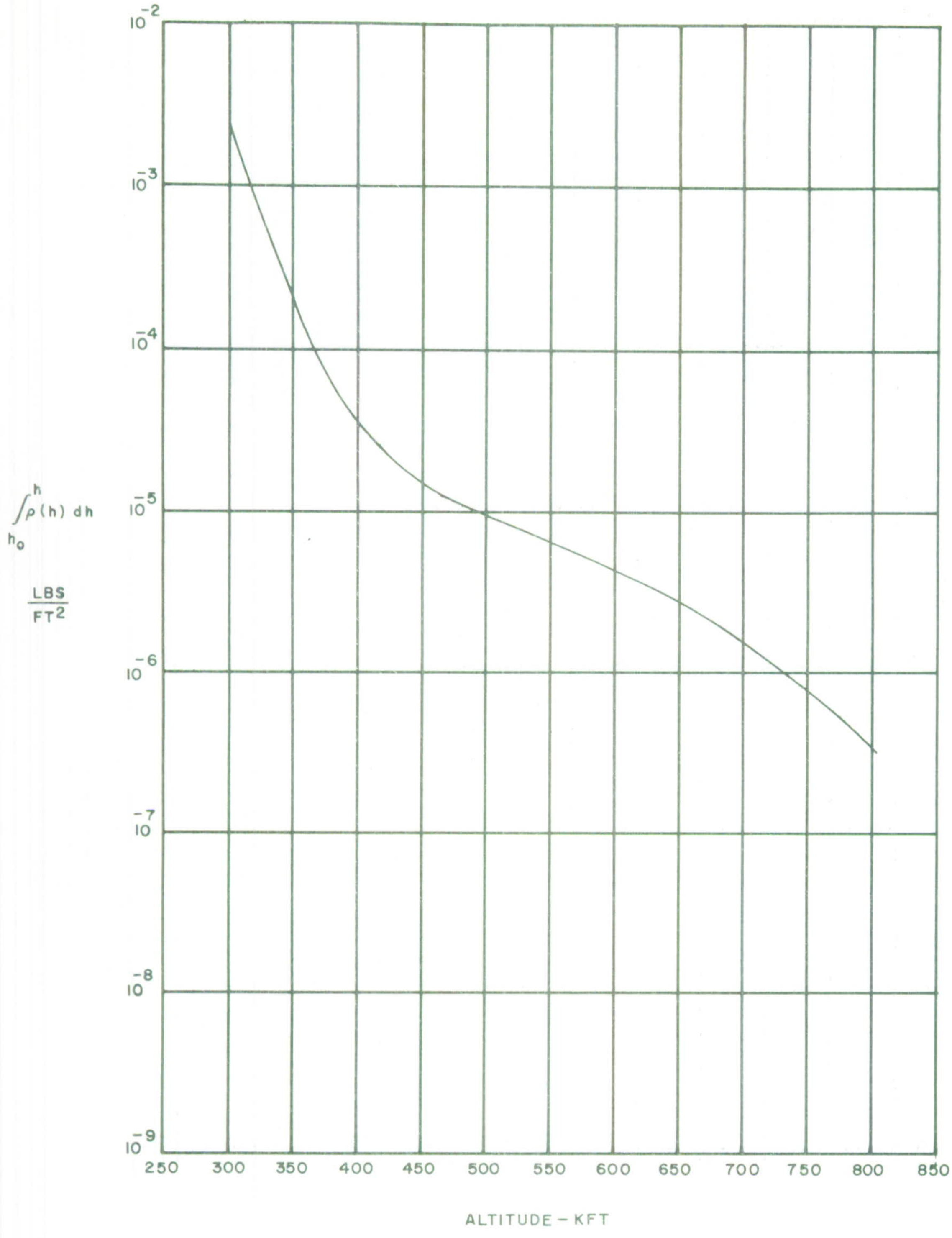

Figure 5

$$\int_{h_0}^{h} \rho(h)\, dh \ \frac{LBS}{FT^2}$$

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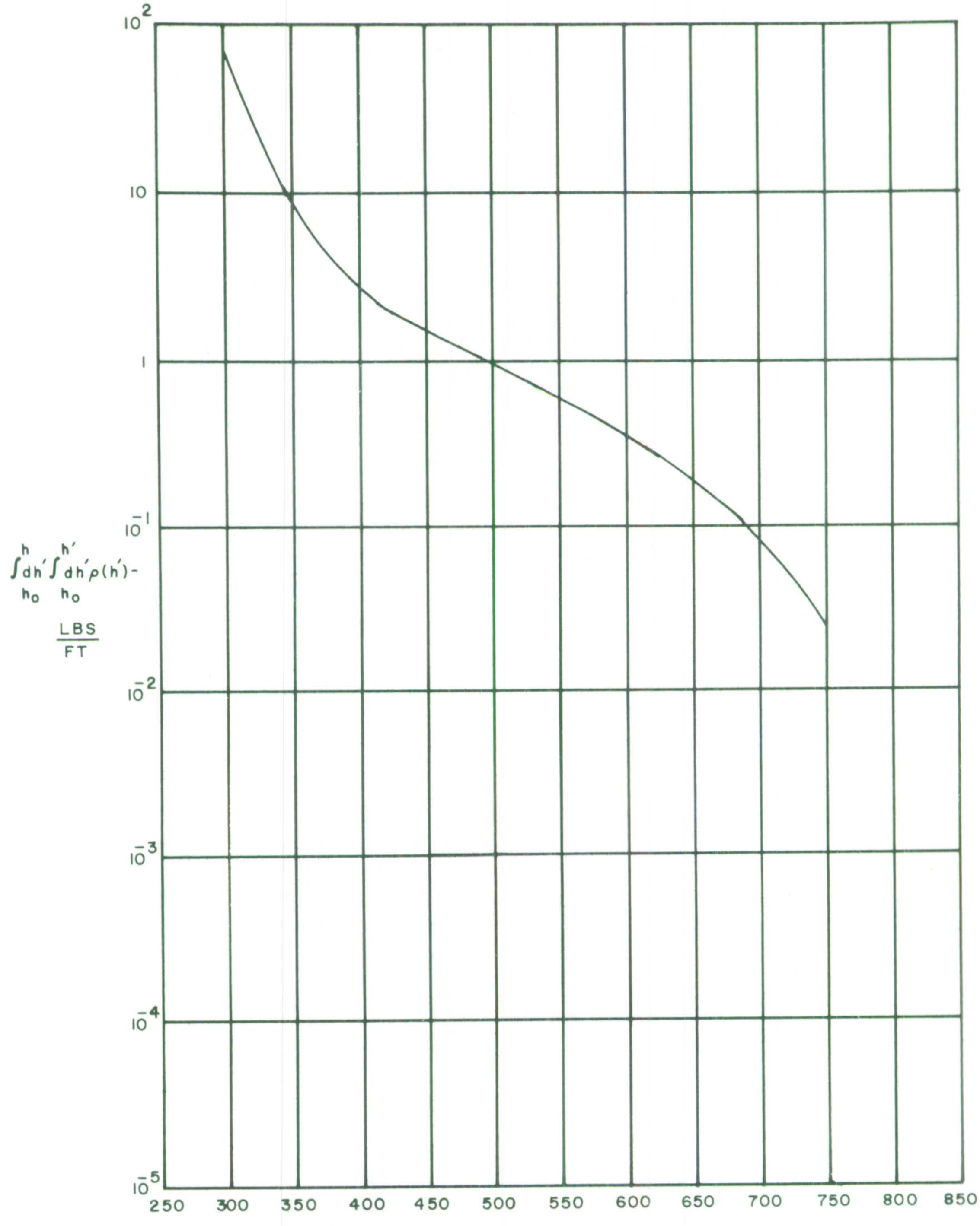

Figure 6

$$\int_{h_o}^{h} dh' \int_{h_o}^{h'} dh' \rho(h') \ \frac{LBS}{FT}$$

IB-17.742

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